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An absorption model with correct Regge cut branch point behaviour

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Abstract. The effect of modifying the absorption model, as suggested by Guerin, to give Regge cuts with the correct branch point behaviour is investigated. The integrals are reduced to a form which makes application of the modified model practicable.

1. Introduction

In recent years the absorption model (Sopkovich 1962, Gottfried and Jackson 1964, Watson 1965) has become widely accepted as a method of adding Regge cut contributions to Regge pole amplitudes for two-body scattering processes. These cuts are usually interpreted as double-Regge exchange contributions, where one of the Regge poles is a pomeron. (In most phenomenological models there is an assumption that the pomeron is a simple Regge pole.) Early reggeized absorption models (see, eg Arnold and Blackmon 1968, Henyey *et al* 1969, Ross *et al* 1970, Adjei *et al* 1972) assumed that the elastic rescattering amplitude was purely imaginary which is equivalent to assuming a fixed pole pomeron. In more recent models (Collins and Swetman 1972) it has been found necessary to give the pomeron trajectory a more realistic slope. Although having many of the properties required of double-Regge cuts (eg correct position of the branch point) the cut terms generated by the absorption model do not have the correct discontinuity near the branch point. The absorption model produces cuts with a logarithmic behaviour at the branch point whereas, according to Gribov *et al* (1965), double-Regge cuts are expected to have an inverse logarithmic form. The cause of the failing is most easily seen by rewriting the absorption corrections in integral form.

We write the partial-wave expansion of an s-channel CM helicity amplitude $A_H(s,t)$ as

$$A_H(s,t) = 16\pi \sum (2j+1) a_{jH}(s) d^j_{\mu\mu'}(z_s), \tag{1}$$

where $H = \{\mu_1, \mu_2, \mu_3, \mu_4\}$ stands for the helicities of the initial (μ_1, μ_2) and final (μ_3, μ_4) state particles, $\mu = \mu_1 - \mu_2$, $\mu' = \mu_3 - \mu_4$ and z_s is the s-channel CM scattering angle.

In its most familiar form the absorption model says that the effect of initial and final state rescatterings is to modify the partial-wave amplitudes, $a_{jH}(s)$, of a simple Regge pole amplitude according to

$$a'_{jH}(s) = a_{jH}(s) + \mathrm{i}\rho^{\mathrm{i}} b^{\mathrm{i}}_j a_{jH}(s) + \mathrm{i} a_{jH}(s) b^{\mathrm{f}}_j \rho^{\mathrm{f}} \tag{2}$$

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where $a'_{jH}(s)$ are the modified partial-wave amplitudes and b^i_j and b^f_j are the partial-wave amplitudes for the initial and final state rescattering which is usually assumed to be s-channel helicity conserving. ρ^i and ρ^f are density of states factors which, assuming pairwise equal-mass scattering are the same for initial and final states and are given by

$$\rho = \rho^i = \rho^f = \frac{2q_s}{\sqrt{s}}, \tag{3}$$

where q_s is the s-channel CM three-momentum. In practice it is usual to assume that b^i and b^f are the same. With this assumption we can rewrite the correction term to the amplitude as

$$A^{cut}_H(s,t) = 32\pi i\rho \sum (2j+1) b_j(s) a_{jH}(s) d^j_{\mu\mu'}(z_s). \tag{4}$$

Using the definitions of a_{jH} and b_j together with the properties of the d^J functions it may be shown that equation (4) may be written (for large s) as (Collins 1971)

$$A^{cut}_H(s,t) = \frac{i}{16\pi^2 q_s\sqrt{s}} \int\int \frac{dt_1\, dt_2}{\sqrt{K}} \theta(K) A_H(s,t_1) B(s,t_2) \cos[(\mu-\mu')\phi], \tag{5}$$

where

$$K(t,t_1,t_2) = 2(tt_1+tt_2+t_1t_2) - t_1^2 - t_2^2 - t^2$$

and

$$B(s,t) = 16\pi \sum (2j+1) b_j(s) d^j_{00}(z_s).$$

ϕ is given by

$$\cos\phi = \frac{t_2 - t_1 - t}{2\sqrt{tt_1}} + O\left(\frac{1}{s}\right).$$

Equation (5) may be compared with the double-Regge exchange contribution obtained by considering the Gribov diagram of figure 1.

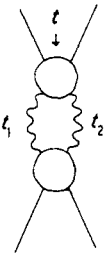

Figure 1. Gribov diagram for double Regge exchange.

$$A^{cut}_{H_t} = \frac{i}{s} \int\int \frac{dt_1\, dt_2}{\sqrt{K}} \theta(K) s^{\alpha_R(t_1)+\alpha_p(t_2)} \xi_R(t_1) \xi_p(t_2) N_{\lambda_1\lambda_3}(t,t_1,t_2) M_{\lambda_2\lambda_4}(t,t_1,t_2), \tag{6}$$

where α_R and α_p are the Regge pole and pomeron trajectory functions, ξ_R and ξ_p are signature factors and N and M are vertex functions which, for the two-reggeon cut, are essentially the fixed-pole residues of the particle–particle → Regge–Regge amplitude. λ_i are t-channel CM helicity labels. The work of Gribov *et al* (1965) and White (1972) indicates that N and M should be evaluated on different sides of the two-reggeon cut.

Comparing equations (5) and (6) we see that the only essential difference is that the absorption model neglects the vertex functions. However in Gribov *et al* (1965) and Collins (1971) it is shown that these functions must themselves contain the two-reggeon branch point and that near this point they should behave as

$$a_{jH_t}, N, M \underset{j\to\alpha_c}{\sim} \frac{1}{\ln(j-\alpha_c)}. \tag{7}$$

Guerin (1972)† has suggested a simple modification to the absorption model which gives the cuts the correct form. This consists of taking account of the vertex structure by introducing an extra factor in equation (5):

$$A_H^{\text{cut}} = \frac{\mathrm{i}}{16\pi^2 q_s\sqrt{s}} \int\int \frac{\mathrm{d}t_1\,\mathrm{d}t_2}{\sqrt{K}} \frac{\theta(K)A_H(s,t_1)B(s,t_2)\cos[(\mu-\mu')\phi]}{[1-\lambda\ln(\alpha_c(t)-\alpha_R(t_1)-\alpha_p(t_2)+1)]^2+\pi^2\lambda^2}, \tag{8}$$

where

$$\alpha_p = 1+\alpha_p' t_2, \qquad \alpha_R = \alpha_0+\alpha_R' t_1, \qquad \alpha_c = \alpha_0+\frac{\alpha_R'\alpha_p'}{\alpha_R'+\alpha_p'}t = \alpha_0+\alpha_c' t$$

and where λ is taken to be a constant. This modification is the simplest form which incorporates the desired properties. For $\lambda = 0$ we regain the traditional absorption model. We believe that such a parametrization may not be too great a simplification as the absorption model has, except for a few well known failures, been successful in fitting the data. It should be noted that the modification to the absorption model adopted in equation (8) makes two assumptions. Firstly, we have assumed that λ is the same for all helicity amplitudes. The factor

$$\frac{1}{[1-\lambda\ln(\alpha_c-\alpha_R-\alpha_p+1)]^2+\pi^2\lambda^2}$$

is to be thought of as a means of incorporating the branch-point structure of the vertex functions N and M in the model. Therefore if λ has any helicity dependence it should be expressed in terms of the t-channel helicity labels whereas the absorption model is written in terms of s-channel helicity amplitudes. This would result in a much more complicated structure for equation (8). The second assumption we have made is that the correction factor is real; this is not strictly necessary except where the t-channel process is elastic.

We shall investigate the consequences of making the above modification to the absorption model. We find that Guerin's conclusions for the case of two parallel trajectories no longer hold when the trajectories have different slopes.

2. Evaluation of the cut contribution

In phenomenological applications equation (8) could be integrated numerically as it stands; however, it is possible to make a considerable simplification of the integral before recourse to the computer is necessary. Making the substitution

$$t = -\boldsymbol{k}^2, \qquad t_1 = -\boldsymbol{q}^2, \qquad t_2 = -(\boldsymbol{k}-\boldsymbol{q})^2, \tag{9}$$

† In this paper, modifications of the kind discussed here are considered for the case of parallel trajectories. In considering the case of non-parallel trajectories the author makes assumptions which limit her to parallel trajectories again.

where $\boldsymbol{k} = (k_1, k_2)$ and $\boldsymbol{q} = (q_1, q_2)$ are two-dimensional vectors, we may write equation (8) as

$$A_H^{\text{cut}} = \frac{\mathrm{i}}{16\pi^2 q_s \sqrt{s}} \int_{-\infty}^{\infty} \mathrm{d}^2 q \frac{A_H(t_1) B(t_2) \cos[(\mu - \mu')\phi]}{(1 - \lambda \ln D)^2 + \pi^2 \lambda^2}, \tag{10}$$

where

$$D = \alpha_c(t) - \alpha_R(t_1) - \alpha_p(t_2) + 1 = \alpha_c' t - \alpha_R' t_1 - \alpha_p' t_2 .$$

Choosing a frame in which $k_1 = 0$ and changing to new coordinates u and v such that

$$u = q_1^2 + \left(q_2 - \frac{\alpha_R' k_2}{\alpha_R' + \alpha_p'} \right)^2$$

$$v = q_2 - \frac{\alpha_R' k_2}{\alpha_R' + \alpha_p'},$$

we obtain

$$A_H^{\text{cut}} = \frac{\mathrm{i}}{16\pi^2 q_s \sqrt{s}} \int_0^{\infty} \mathrm{d}u \int_{-\sqrt{u}}^{\sqrt{u}} \mathrm{d}v \frac{A_H(t_1) B(t_2) \cos[(\mu - \mu')\phi]}{(u - v^2)^{1/2} [(1 - \lambda \ln D)^2 + \pi^2 \lambda^2]}, \tag{11}$$

where

$$t_1 = -u - 2hv - h^2$$

$$t_2 = -u - 2hv - h^2 + t + 2k_2(v + h)$$

$$D = (\alpha_R' + \alpha_p')u$$

$$h = \frac{\alpha_R' k_2}{\alpha_R' + \alpha_p'}.$$

In order to illustrate the effect of modifying the absorption model we shall follow Guerin and take a simple parametrization for the Regge pole and pomeron amplitudes for a $0^- \tfrac{1}{2}^+ \to 0^- \tfrac{1}{2}^+$ process

$$\begin{aligned} B(t_2) &= -n \exp(-\tfrac{1}{2} \mathrm{i} \pi \alpha_p) \exp(d t_2) s^{\alpha_p} \\ A(t_1)_{\text{non-flip}} &= p[1 + \tau \exp(-\mathrm{i}\pi\alpha_R)] \exp(c t_1) s^{\alpha_R} \\ A(t_1)_{\text{flip}} &= r\sqrt{-t_1}[1 + \tau \exp(-\mathrm{i}\pi\alpha_R)] \exp(f t_1) s^{\alpha_R}, \end{aligned} \tag{12}$$

where n, p, r, c, d and f are constants and τ is the signature of the Regge pole. Substituting equations (12) in equation (11) we see that all the integrals may be written in the form

$$Q(Y, Z, \lambda) = \int_0^{\infty} \mathrm{d}u \int_{-\sqrt{u}}^{\sqrt{u}} \mathrm{d}v \frac{\exp[Y t_1(u, v) + Z t_2(u, v)]}{(u - v^2)^{1/2} [(1 - \ln D)^2 + \pi^2 \lambda^2]} \tag{13}$$

or as derivatives of Q with respect to X or Y. The integration over v in equation (13) may be carried out to obtain

$$\begin{aligned} Q(Y, Z, \lambda) = \pi \exp\left[\left(\frac{\alpha_R'^2 Y + \alpha_p'^2 Z}{(\alpha_R' + \alpha_p')^2} \right) t \right] \\ \times \int_0^1 \mathrm{d}x \frac{\exp[(Y + Z - 1) \ln x] I_0\{2[(Y\alpha_R' - Z\alpha_p')/(\alpha_R' + \alpha_p')](t \ln x)^{1/2}\}}{\{1 - \lambda \ln[-\ln x(\alpha_R' + \alpha_p')]\}^2 + \pi^2 \lambda^2}, \end{aligned} \tag{14}$$

where I_0 is a modified Bessel function and where we have made the additional substitution $u = -\ln(x)$. In terms of Q the non-flip cut becomes

$$A^{\text{cut}}_{\text{non-flip}} = \frac{-nps^{\alpha_0+1/2}}{16\pi^2 q_s}[Q(c+\alpha'_R \ln s, d+\alpha'_p \ln s - \tfrac{1}{2}i\pi\alpha'_p, \lambda) + \tau \exp(-i\pi\alpha_0) Q(c+\alpha'_R \ln s - i\pi\alpha'_R, d+\alpha'_p \ln s - \tfrac{1}{2}i\pi\alpha'_p, \lambda)]. \quad (15)$$

The single integral of equation (15) may be integrated numerically at speeds far greater than those needed for the double integral of equation (10) thus making practical application of the modified model feasible.

In order to illustrate the effects of the above modification to the absorption model we have evaluated the non-flip absorption correction for a typical positive and negative signature pole amplitude. Consider, for example, the K^* and K^{**} contributions to the non-flip amplitudes for hypercharge exchange processes. We may parametrize these as

$$A_{K^*}(s,t) = p(1-e^{-i\pi\alpha}) e^{ct} s^{\alpha}$$
$$A_{K^{**}}(s,t) = p(1+e^{-i\pi\alpha}) e^{ct} s^{\alpha}, \quad (16)$$

where we have assumed the K^* and K^{**} to be exchange degenerate. We take the rescattering amplitude to be

$$B(s,t) = -n \exp(-\tfrac{1}{2}i\pi\alpha_p) \exp(dt) s^{\alpha_p} \quad (17)$$

with $\alpha_p = 1+0{\cdot}35t$. Finally we take

$$c+\alpha'_R \ln s = 4, \qquad d+\alpha'_p \ln s = 3{\cdot}5$$

which correspond to typical values (Giacomelli 1972) of 8 GeV^{-2} and 7 GeV^{-2} for the slope of the forward peaks of hypercharge exchange ($\pi N \to K\Sigma$, $\bar{K}N \to \pi\Sigma$) and πN elastic differential cross sections at 5 GeV/c. The absorption correction terms for different values of λ obtained with these amplitudes are shown in figures 2–5, where the normalization is arbitrary.

3. Discussion

From figures 2–5 we can see that the effects of making the proposed modifications to the absorption model are quite considerable. For both the K^* and K^{**} amplitudes the magnitudes of the absorption corrections vary considerably with λ. For positive λ the 'cut' contributions are less than those for the absorption model but for small negative values of λ ($\lambda \gtrsim -0{\cdot}3$) we find that the cut contributions are enhanced and that, as stated by Guerin, the ratio $\Lambda = |A_{\text{cut}}(\lambda)|/|A_{\text{cut}}(\lambda=0)|$ is approximately independent of t. Λ is found to have a maximum value of about 2. Thus we see that our modifications to the absorption model can produce the kind of enhancement of absorption corrections made in the 'strong cut' absorption model. For large λ Λ falls below 1.

If we now look at the effect of our modifications on the phases of the cut contributions we see that for the K^{**} amplitude the phase of the absorption correction is little changed. However, for the K^* trajectory the phase at wide angles changes considerably with λ. For positive λ the phase is smaller than for the traditional absorption model but for the small negative values where $\Lambda > 1$ we find that the phase is greater than that for $\lambda = 0$. This means that the absorption of the imaginary part of the pole amplitude is increased relative to that of the real part.

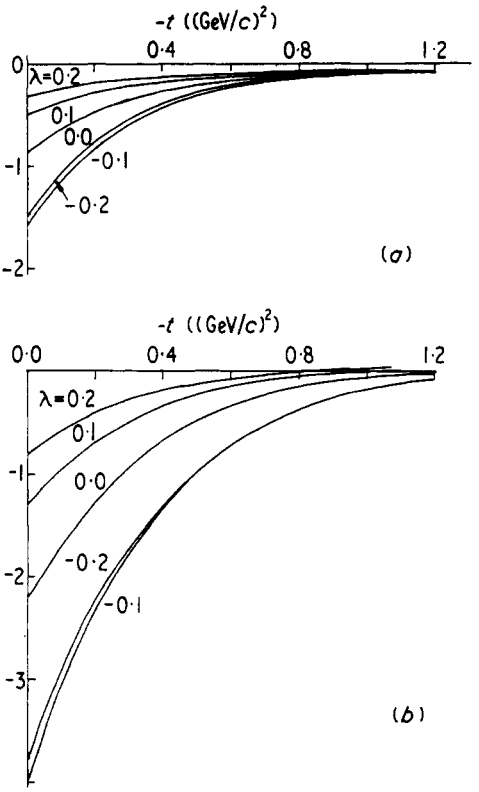

Figure 2. Real and imaginary parts of the absorption correction to the non-flip K^* amplitude. (*a*) Re $A^{cut}_{++}(K^*)$; (*b*) Im $A^{cut}_{++}(K^*)$.

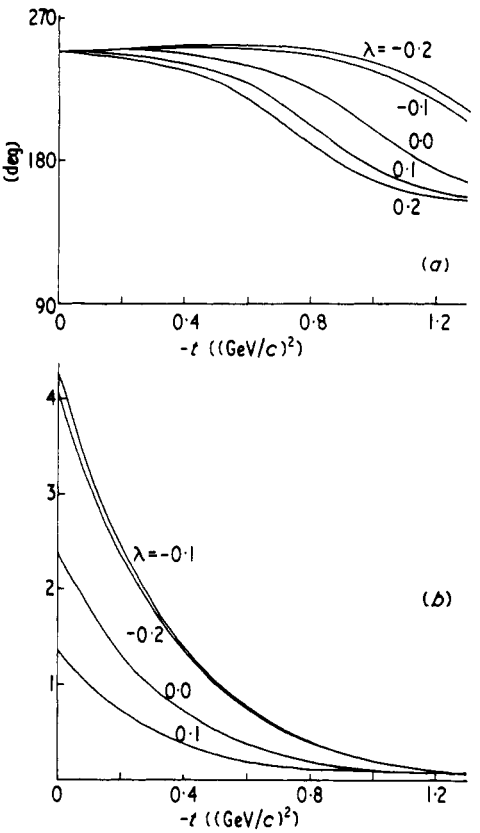

Figure 3. Modulus and phase of the absorption correction to the non-flip K^* amplitude. (*a*) Phase $A^{cut}_{++}(K^*)$; (*b*) modulus $A^{cut}_{++}(K^*)$.

In several recent phenomenological papers (see, eg, Ringland *et al* 1972, Sadoulet 1972, Barkai and Moriarty 1972) models in which the imaginary parts of pole amplitudes are more strongly absorbed than the real parts have been proposed in order to explain the cross-over effects in elastic differential cross sections. We do not feel that the modifications to the absorption model proposed here provide a justification for such models since, in order to explain cross overs, these models require a 'cut contribution' phase at $t \sim -0{\cdot}15$ considerably different from that given by the absorption model. Our modifications do not significantly alter the phase of the absorption corrections until $|t| \gtrsim 0{\cdot}5$. However, it is worth noting that Collins and Swetman (1972) have shown that, if the elastic rescattering amplitude is given the correct phase, it is possible to explain the πN scattering data for $|t| < 0{\cdot}5$ using the traditional absorption model with the absorption corrections enhanced by a factor of 1·48–1·71. At wider angles their model does not fit the data and it is possible that this is due to the need for modifications to the absorption model of the kind we have described.

Our calculation differs from that of Guerin in that we have taken account of the different t dependence of the phases of the K^*, K^{**} and pomeron pole amplitudes. As a consequence, in contrast to Guerin's general statement that the change in the phase of the cut contribution is small, the modification to the absorption model considered here can result in a substantial change in the phase of the $K^* - P$ cut contribution.

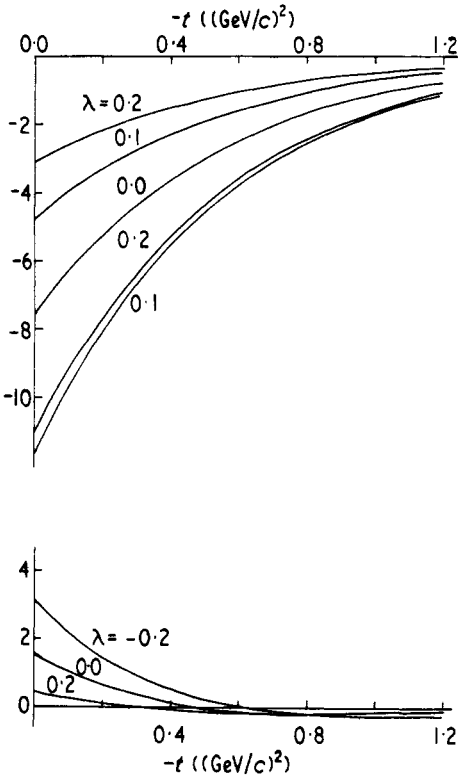

Figure 4. Real and imaginary parts of the absorption correction to the non-flip K^{**} amplitude. (*a*) Re $A^{cut}_{++}(K^{**})$; (*b*) Im $A^{cut}_{++}(K^{**})$.

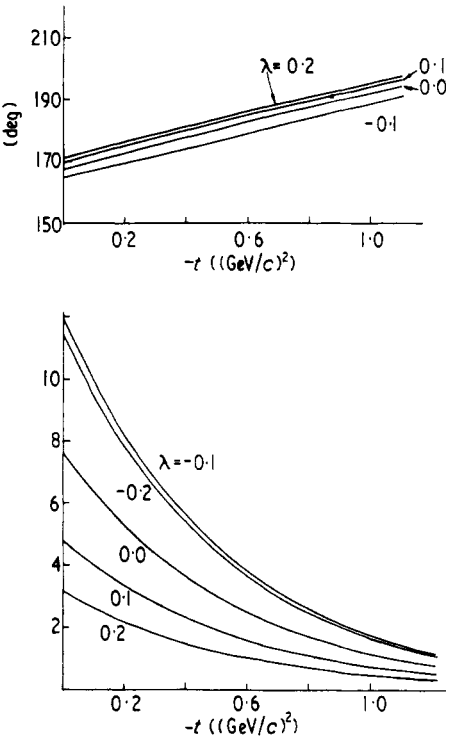

Figure 5. Modulus and phase of the absorption correction to the non-flip K^{**} amplitude. (*a*) Phase $A^{cut}_{++}(K^{**})$; (*b*) modulus $A^{cut}_{++}(K^{**})$.

We have seen how, as a result of modifying the absorption model to give the cuts the correct double-Regge discontinuity behaviour at the branch point, an enhancement of absorption corrections of the type made in many phenomenological models may arise. We have also seen that at wide angles the phase of the absorption corrections may also be altered considerably. Many simplifications have been made in the particular modification to the absorption model we have considered. However, we feel that by making such theoretical improvements to the absorption model many of the *ad hoc* modifications made in phenomenological applications may have a natural explanation.

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